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This final report briefly describes research results on a theory of gain scheduling for flight control applications that were obtained by the Principal Investigator and his students over the two year period of support. Results reported include the development of a solution to an input-output pseudolinearization problem for nonlinear systems, characterzation of the impact of linear controller configuration on the gain scheduling process, and formulation of an approach to gain scheduling in the face of rapidly-varying scheduling signals. Publications describing the results in detail are listed.

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GAIN SCHEDULING FOR ROBUST LINEAR CONTROLLERS

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1. Research Objectives

The overall research objective was to further develop recent advances in a theory of gain scheduling as a design methodology for nonlinear flight control systems. A particular objective was to investigate the interaction of gain scheduling requirements with features of underlying linear controller designs based on modern robust linear control methods. This leads to linear controller architectures that facilitate the subsequent gain scheduling phase of the overall design. A second objective was to devise gain scheduling techniques leading to controllers that maintain good performance characteristics in the face of rapidly-varying scheduling signals.

2. Research Status

The impact of linear controller configuration on the process of gain scheduling and the performance of the resulting controller is substantial. This was demonstrated in our earlier work on a simple flight control model,[1] and we have devoted considerable effort to clarifying the situation in a general theoretical framework. The results, presented in publication [4], can be described briefly as follows. For a general nonlinear plant and general linearized controller architecture, we derived conditions that are necessary and sufficient for the absence of *hidden coupling terms* in the resulting gain scheduled design. (This is our formal name for the phenomenon of unintended coupling of variables that has long been recognized in practice as a limiting aspect of gain scheduled controllers.) These conditions can be interpreted and applied in several ways. Proposed linear controller architectures can be assessed in terms of satisfying these conditions. In this regard integrator components in the controller play a key role, a situation that is well understood in practice, at least in a single-input, single-output setting. However there seems never to have been a study of this issue in a general setting. Perhaps the most surprising result is that 'linearly equivalent' linear controller configurations involving integrators can have different effects on the gain scheduling problem. That is, by rearranging the linear controller design in a way that does not change the linear controller behavior it is possible that the resulting scheduled controller can exhibit different numbers of deleterious hidden coupling terms. This is illustrated in an example in [4]. Another contribution is that the gain scheduling procedure we propose in conjunction with our conditions yields flexibility to assign unavoidable coupling to relatively innocuous variables. Another application is the use of our conditions to analyze proposed, ad-hoc scheduling schemes for potential performance-limiting characteristics. Indeed we have carried out such an analysis for a particular autopilot structure in a related project described in Section 5, the core features of which are presented in another example in [4].

Because the foundation of our approach to gain scheduling involves basic ideas of extended linearization and pseudolinearization, another part of our activity was in contributing to the knowledge base in these areas. Publication [2] presents an approach to input-output pseudolinearization for multi-input, multi-output nonlinear systems. This is the problem of applying feedback and a coordinate change to a nonlinear plant with the objective of obtaining a closed-loop description with the following property: The input-output behavior of the family of closed-loop linearizations about constant operating points is independent of the operating point. Necessary and sufficient conditions for existence of a solution to this problem were developed, and

[1] R.A. Nichols, R.T. Reichert, and W.J. Rugh, "Gain Scheduling H-Infinity Controllers: A Flight Control Example," *IEEE Transactions on Control Systems Technology*, Vol. 1, No. 2, pp. 69 - 75, 1993.

applications to nonlinear control problems, including an approximate tracking problem, were explored. This work also provides a method of proof of the original multi-input pseudolinearization result (with no specified output) announced over a decade ago, though no proof ever appeared in the archival literature.

Additional research accomplishments focus on developing an approach for gain scheduling in the face of rapidly-varying scheduling signals. It is widely recognized that performance deterioration of gain scheduled controllers as the scheduling signal varies more rapidly is a limitation of the approach. The most promising avenue involves modifications to the basic theory of output regulation with exogenous signals. Recent advances in that theory have been based on the assumption of an exogenous system that generates the exogenous signals, an assumption that is inappropriate in the setting of gain scheduling. (Scheduling signals are in fact generated by other parts of the overall system, but the scheduling formulation is adopted precisely because of ignorance about, or complexity of, the overall system.) Thus our efforts focused on the extent to which satisfactory results can be obtained without such an assumption. Detailed results are found in [1], [5], and [6], and can be described briefly as follows.

In [1] we consider design of a gain scheduled control law for a nonlinear plant with the objective of regulating the output to zero in the face of a rapidly-varying scheduling signal. The basic idea is the derivative provides more information about the scheduling signal, beyond just the instantaneous value, and this information can be used to reduce performance deterioration. Based on an underlying theoretical construct called an approximate regulation manifold, a state feedback control law that depends on the instantaneous values of the scheduling signal and its time derivative were designed to meet the objective under the assumption that the scheduling signal is slowly accelerating, i.e., has small second derivative.

Because of the stringent assumptions required in [1], we took a somewhat different approach to the problem in [5]. In this paper a weaker form of approximate regulation manifold is defined, and notions of ultimate boundedness are used to define the nature of approximate regulation. This new formulation enabled necessary and sufficient conditions to be established for the existence of a scheduled control law that meets the objectives by making use of the first derivative of the scheduling signal. These results have been illustrated on a number of examples, and the performance improvement over a gain scheduled design that does not use the scheduling signal derivative is substantial. An interesting spin-off of this work is a new stability criterion for linear parameter-varying systems. [3]

While it is often feasible in applications to consider using the first derivative of a scheduling signal, the use of higher-order scheduling signal derivatives (corresponding to using even more information about the scheduling signal) is problematic because of noise. A discrete-time setting avoids this problem since using higher-order differences of a discrete-time scheduling signal corresponds to using past values of the signal. Thus a general version of our results was developed for the discrete-time case in [6]. Again ultimate boundedness characterizations were used. Necessary and sufficient conditions were obtained for existence of a solution to the approximate regulation problem using a controller that depends on the current and (a fixed number of) past values of the scheduling signals. The corresponding regulation error is ultimately bounded as a function of the corresponding highest difference of the scheduling signal. Again performance improvements over controllers using only the current values of the scheduling signal were significant. The next step in pursuing this direction of research is to develop the results for dynamic output feedback—a much more practical case than the static state feedback case considered to date.

3. Publications

1. D. Guo and W.J. Rugh, "On Output Regulation Problems Involving Exogenous Signals," *Proceedings of the 1993 American Control Conference,* pp. 2951-2955, San Francisco, CA, June 1993.
2. D.A. Lawrence and W.J. Rugh, "Input-Output Pseudolinearization for Nonlinear Systems," *IEEE Transactions on Automatic Control,* Vol. 39, No. 11, pp. 2207 - 2218, 1994
3. D. Guo and W.J. Rugh, "A Stability Result for Linear Parameter-Varying Systems," *Systems & Control Letters,* Vol. 24, pp. 1 - 5, 1995
4. D.A. Lawrence and W.J. Rugh, "Gain Scheduling Linear Dynamic Controllers for a Nonlinear Plant," *Automatica,* Vol. 31, No. 3, pp. 381 - 390, 1995. (Preliminary version in *Proceedings of the 1993 IEEE Conference on Decision and Control,* pp. 1024-1029, San Antonio, TX, December, 1993.)
5. N. Sureshbabu and W.J. Rugh, "Output Regulation with Derivative Information," *Proceedings of the 1994 IEEE Conference on Decision and Control,* pp. 2356 - 2361, Orlando, FL, December, 1994. (Final version accepted for publication in *IEEE Transactions on Automatic Control,* 1995.)
6. N. Sureshbabu and W.J. Rugh, "On Output Regulation for Discrete-Time Nonlinear Systems," *Proceedings of the 1995 American Control Conference,* pp. 4226 - 4231, Seattle, WA, June, 1995 (Also to be submitted for journal publication.)

4. Personnel

Principal Investigator:

Wilson J. Rugh

Research Assistants:

N. Sureshbabu: PhD completed in December, 1994. Dissertation Title: *On Output Regulation of Nonlinear Systems Using Derivative-Difference Information*

D. Guo: former graduate student.

5. Interactions

Publications [1], [4], [5], and [6] were presented at the cited conferences. Both [1] and [6] won "Best Presentation in Session" awards at the respective American Control Conferences. Also related to our research effort, we co-organized (with P.P. Khargonekar) a special session on Gain Scheduling at the *1993 IEEE Conference on Decision and Control.* This session brought together researchers working on a variety of aspects of gain scheduling, from theory to practice. Two indicators of the success of the session are that in the review process our session proposal was ranked first out of 28 proposals received by the Program Committee, and in the actual session the room was often over-crowded with conference attendees struggling to hear from the hallway.

A second related activity is the application of our theory to a missile autopilot project at the Johns Hopkins Applied Physics Laboratory (APL). We performed a preliminary evaluation of a

proposed linear autopilot configuration and gain scheduling procedure. The objective was to assess potential problems in scheduling that could arise, and suggest a scheduling approach to circumvent the problems. Indeed we concluded that the proposed procedure was suspect in that a large number of hidden coupling terms occur in the final controller. We have suggested an alternate scheduling procedure, and confirmed its advantages in a single-axis simulation.[2] This activity is preliminary to an "Advanced Technology Demonstration" project at APL involving the design of high-performance autopilots and leading to flight tests on a tactical missile. Our interaction with this project will continue to provide both motivation and application evaluation for our research effort.

[2] W.J. Rugh and P.B. Jackson, "Analysis of Gain Scheduling for the Three-Loop Autopilot Structure," Technical Report JHU/ECE 94-02, Department of Electrical and Computer Engineering, Johns Hopkins University, Baltimore, MD, 1994.